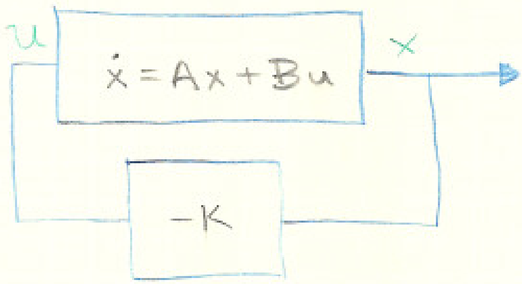

$u = -Kx$

Adjust this matrix to get desired results.

$$\therefore \quad \dot{x} = Ax - Bkx = \overbrace{(A - Bk)}^{\bar{A}}x$$

$$\mathcal{C} = [B \quad AB \quad A^2B \; \dots\dots \; A^{n-1}B]$$

if rank$(\mathcal{C}) = n$, then the system is state controllable.

if k such that eig$(A - Bk) = \begin{bmatrix} u_1 \\ \vdots \\ u_2 \end{bmatrix}$

TEST OF RANK

See notes from last day.

$$\dot{x} = Ax + Bu \qquad x \in \mathbb{R}^n \quad A \in \mathbb{R}^{n \times n}$$

$$y = Cx \qquad u \in \mathbb{R} \quad B \in \mathbb{R}^{n \times m}$$

$$\underset{n \times mn}{\mathcal{C}} = [\underset{n \times m}{B} \quad \underset{n \times m}{AB} \; \dots\dots \; \underset{n \times m}{A^{n-1}B}]$$

Observability matrix.

$$\dot{x} = Ax + Bu \qquad y \in \mathbb{R}^r$$
$$y = Cx \qquad C \in \mathbb{R}^{r \times n}$$

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \begin{matrix} \leftarrow r \times n \\ \leftarrow r \times n \\ \\ \leftarrow r \times n \end{matrix} \Bigg\} \; nr \times n$$

$$\underset{n \times nr}{\mathcal{O}^T} \cdot \underset{nr \times n}{\mathcal{O}} = \underset{n \times n}{[\quad]}$$

Ex:
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u.$$

determine if this matrix is controllable.

$$\mathcal{C} = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$\det(\mathcal{C}) = 0 \quad \therefore \quad \operatorname{rank}(\mathcal{C}) \neq n \;\therefore$ not completely state controllable.

Ex:
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u.$$

$\mathcal{C} = [B \;\; AB] = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \quad \det(\mathcal{C}) = -1 \;\therefore\; \operatorname{rank}(\mathcal{C}) = n$

$\therefore$ this system is completely state controllable.

Ex:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\mathcal{C}_1 = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -2 & -1 \\ 1 & 1 & -2 & 1 & 2 & 1 \end{bmatrix}$$

$$\qquad\qquad\qquad\qquad\quad \underbrace{}_{B} \quad \underbrace{}_{AB} \quad \underbrace{}_{A^2B}$$

$$\mathcal{C}_1 \cdot \mathcal{C}_1^T = \begin{bmatrix} 3 & -3 & 4 \\ -3 & 7 & -8 \\ 4 & -8 & 12 \end{bmatrix}$$

$\det(\mathcal{C}) = 32 \therefore \text{rank}(\mathcal{C}) = n$

$\therefore$ the system is completely state controllable.

Ex: example of pole placement.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

design a state feedback $u = -Kx$ to acheive the following performance.

* $x_1(t)$ and $x_2(t)$ bounded for all $x_1(0)$ & $x_2(0)$
* $x_1(t) \to 0$ $\quad t \to \infty$ $\qquad x_2(t) \to 0$ $\quad t \to \infty$

$\dot{x} = Ax + Bu$

$u = -Kk$

$$\dot{x} = (A - BK)x \quad \xRightarrow{\mathcal{L}^{-1}} \quad x(t) = e^{(A-BK)t}\, x(0)$$

if K is such that the eigen values of $A - BK$ are in the LHP, then $x(t)$ is bounded and $x(t) \to 0, t \to \infty$.

$\therefore$ move poles into LHP.

so let's determine T_s, PO as well.

the eigen values of the CL system are given by.

$$\det(\lambda I - (A - BK))$$

$$\lambda I - (A - BK) = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} - \left(\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2-k_1 & -1-k_2 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda_1 - 1 & 1 \\ k_1 - 2 & \lambda_2 + 1 + k_2 \end{bmatrix}$$

for $\det(\lambda I - (A - BK))$, then

$$(\lambda_1 - 1)(\lambda_2 + 1 + k_2) - (k_1 - 2) = 0$$

$$\lambda^2 + \lambda + \lambda k_2 - \lambda - 1 - k_2 - k_1 - 2 = 0$$

$$\underbrace{\lambda^2}_{a} + \underbrace{(k_2)}_{b}\lambda - \underbrace{(3 - k_2 - k_1)}_{c} = 0 \qquad (1)$$

$$\lambda^2 + 2\zeta\omega_n \lambda + \omega_n^2 \qquad (2)$$

combine (1) and (2) to get desired T_s, PO.

Ex:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$u = -kx \qquad k = [k_1 \ \ k_2 \ \ k_3]$$

$$\det(\lambda I - (A - Bk)) = 0$$

$$\Rightarrow \quad a\lambda^3 + b\lambda^2 + c\lambda + d = 0 \qquad ①$$

$$(\lambda + P)(\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2) \qquad ②$$

$$\boxed{P > 5\zeta\omega_n.}$$